# AN ANALOG COMPUTER FOR FREQUENCY RESPONSE STUDIES

By

**FLOYD MARTIN GARDNER**

B. S., Illinois Institute of Technology, 1950
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FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN ELECTRICAL ENGINEERING
IN THE GRADUATE COLLEGE OF THE
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# AN ANALOG COMPUTER FOR FREQUENCY RESPONSE STUDIES

The usual lumped-constant electrical network is described by a rational function, $f(p) = P/Q$ where P and Q are polynomials and p is the complex Laplace variable or "complex frequency". This thesis describes an analog computer intended for the determination of the frequency response, $f(jw)$, of a network. That is, the function $f(p)$ is set into the computer and the frequency response of the computer, $f(jw)$, is measured. The variable w is the angular frequency of the input voltage that is used for the measurements or, alternatively, the imaginary part of p.

A network (the so-called differentiating amplifier) can be constructed whose transfer function is approximately p. Cascading k such networks results in a transfer function of $p^k$. By taking the output from each individual p network and multiplying it a real constant $a^{(k)}$ (this is done with a potentiometer) one obtains a number of terms of the form $a^{(k)}p^k$. These terms are then added together and the sum is a polynomial. Thus the overall transfer function is, to the limit of accuracy of the terms and the summing process, a polynomial P.

In order to realize a transfer function that is the reciprocal

of a polynomial, a P network is placed in the feedback circuit of an amplifier. If the amplifier has a gain of -1, then the overall response is $-1/(1+P)$ or, letting $Q = (1+P)$, then the response is $-1/Q$. A simple phase reversal gives $1/Q$. Only one computer was built so that only P or $1/Q$ could be measured — not $P/Q$. To measure the last would require two computers in cascade.

It was found that the highest degree equation that could be handled was a cubic in p. The noise generated in the first stage was greatly amplified by the following stages so that the noise output of a fourth differentiator (the number of differentiators is the same as the degree of the equation) was much too large to permit measurements on a quartic or higher degree function. The noise output of the third differentiator is sufficiently low so that a cubic can be handled.

The response of the computer was measured over a frequency range of two decades. The actual measurement frequencies used were from 17 to 1700 cps but, for convenience, they were normalized so that the results are shown on a computer (normalized) angular frequency range of 0.1 to 10 radians per second.

Several measured curves of different functions are shown. They include polynomials and their reciprocals and both quadratic and cubic functions. The average observed accuracy of measurement was within 10 per cent of the theoretical values but larger discrepancies were encountered in isolated spots.

The known inaccuracies of construction of the computer are advanced as explanations of the errors.

Investigations of stability were made with the 1/Q network. When Q was a quadratic in p, the results corresponded very closely to theory. However, when Q was made a cubic in p, there were many borderline functions set into the computer that should have caused instability but did not do so. It is believed that this behavior is due to the same inaccuracies that cause errors in the measurement of frequency response.

A simple method of improving the response of a differentiating amplifier is shown in an appendix.

# VITA

Floyd Martin Gardner was born on October 20, 1929 in Chicago, Illinois. He entered the Illinois Institute of Technology in 1946 and received the degree of Bachelor of Science from there in June 1950. He then entered Stanford University and received a degree of Master of Science in June 1951. After leaving Stanford, he came to the University of Illinois. He held Research Assistantships, in the Bureau of Educational Research from June 1951 to February 1952 and in the Electrical Engineering department from February 1952 to June 1952, and a Research Associateship in the Bureau of Educational Research from June 1952 to August 1953.